TEACHING PLAN (SYNOPSIS)

Month : January Subject : Mathematics

TOPIC : Central differences interpolation Paper : 6 (A), Numerical methods

Hours Required	06
Learning Objectives	Guass forward
Previous Knowledge to be reminded	mathematical Calculation
Topic Synopsis	Newton Gauss, Backward

Unit-2

Central difference interpolation Formulae

Theorem:- Gauss's forward formula

If $y=f(x)$ is a function, which takes the values

$y_{-2}, y_{-1}, y_0, y_1, y_2$ corresponding the values of

$x = x_0-2h, x_0-h, x_0, x_0+h, x_0+2h$ then

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}$$

$$\Delta^4 y_{-2} + \text{------} \quad \text{where } u = \frac{x-x_0}{h}$$

Proof:- The newton's forward difference formula is given by

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \text{---} \quad (1)$$

we know $\Delta^2 y_0 - \Delta^2 y_{-1} = \Delta^3 y_{-1} \Rightarrow \Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$

similarly $\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$, $\Delta^4 y_0 = \Delta^4 y_{-1} + \Delta^5 y_{-1}$ --- etc

Also $\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$ and $\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$ etc

Thrust areas	
Skill to be learnt by Student	Calculate the difference table and Identity
Examples/Illustrations	S.chand.
Additional Inputs	Gauss formula

Teaching Models used	Mathematical model
Teaching Aids used	Black-board
References cited	Ref By N. krishnamurthy.
Student Activity planned after the teaching	Assinment, seminar.
Activity planned outside classes	Quiz
Any other	Group discussion

Substituting the values of $\Delta^2 y_0, \Delta^3 y_0, \Delta^4 y_0$ --- in (1), we get

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}(\Delta^2 y_{-1} + \Delta^3 y_{-1}) + \frac{u(u-1)(u-2)}{3!}(\Delta^3 y_{-1} + \Delta^4 y_{-1})$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}(\Delta^4 y_{-1} + \Delta^5 y_{-1}) + \cdots$$

$$= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-1} + \cdots$$

$$= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}[\Delta^4 y_{-2} + \Delta^4 y_{-3}] \cdots$$

$$= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \cdots$$

Theorem:- 2.1.2 (Gauss's Backward formula for Equal intervals)

$y = f(x)$ is a function, which takes the values

$y_{-2}, y_{-1}, y_0, y_1, y_2$ ---- corresponding to the values of

$x = x_0 - 2h, x_0 - h, x_0, x_0 + h, x_0 + 2h$ ---- then

$$y_u = y_0 + u\Delta y_{-1} + \frac{(u+1)u}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-2} + \frac{(u+2)(u+1)u(u-1)}{4!}\Delta^4 y_{-2}$$

Proof:- The Newton's forward difference formula is given by

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \quad (1)$$

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TEACHING PLAN (SYNOPSIS)

Month : Jaunary | Subject : Mathematics

TOPIC : | Paper : 6 (A)

Hours Required	18
Learning Objectives	Guass forward
Previous Knowledge to be reminded	mathematics Calculation
Topic Synopsis	Newton forward, Newton Backward formulas

we have $\Delta y_0 = \Delta y_{-1} + \Delta^2 y_{-1}$ —— ②

$\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$ —— ③

$\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$ --- ④ etc

also $\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$ —— ⑤

$\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$ -- ⑥

with help of ① to ⑥ we get

$$y_u = y_0 + u(\Delta y_{-1} + \Delta^2 y_{-1}) + \frac{u(u-1)}{2!}(\Delta^2 y_{-1} + \Delta^3 y_{-1}) +$$

$$\frac{u(u-1)(u-2)}{3!}(\Delta^3 y_{-1} + \Delta^4 y_{-1}) + \frac{u(u-1)(u-2)(u-3)}{4!}[\Delta^4 y_{-1} + \Delta^5 y_{-1}]$$

$$\Rightarrow y_u = y_0 + u\Delta y_{-1} + \frac{(u+1)u}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} +$$

$$\frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-1} + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_{-1}$$

$$= y_0 + u\Delta y_{-1} + \frac{(u+1)u}{2!} y_{-1} + \frac{(u+1)u(u-1)}{3!}(\Delta^3 y_{-2} + \Delta^4 y_{-2}) +$$

$$\frac{(u+1)u(u-1)(u-2)}{4!}(\Delta^4 y_{-2} + \Delta^5 y_{-2}) + \text{-----}$$

$$y_u = y_0 + u\Delta y_{-1} + \frac{(u+1)u}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-2} + \frac{(u+2)(u+1)u(u-1)}{4!}\Delta^4 y_{-2} + \text{---}$$

Thrust areas	
Skill to be learnt by Student	
Examples/Illustrations	S. chand
Additional Inputs	Guass forward formula

Teaching Models used	mathematics model
Teaching Aids used	Blackboard and chalk
References cited	Ref by V. Venkateswara Rao, N. Krishna murthy, B.V.S.S. Sarma, S. Anjaneya sastry
Student Activity planned after the teaching	Assignment, seminar, viva,
Activity planned outside classes	Quiz
Any other	Group discussion

Solved problems

1) use Gauss's forward formula to find y_{30} for the following data.

$y_{21} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$, $y_{37} = 15.5154$

Sol:

x	21	25	29	33	37
y	18.4708	17.8144	17.1070	16.3432	15.5154

Let us take origin at $x = 29$ & $h = 4$ as the unit.

To find the value of y at $x = 30$, $u = \frac{30-29}{4} = 0.25$

x	u	y_u	Δy_u	$\Delta^2 y_u$	$\Delta^3 y_u$	$\Delta^4 y_u$
21	-2	18.4708				
			-0.6564			
25	-1	17.8144		-0.0510		
			-0.7074		-0.0054	
29	0	17.1070		-0.0564		-0.0022
			-0.7638		-0.0076	
33	1	16.3432		-0.0640		
			-0.8278			
37	2	15.5154				

Putting these values in Gauss forward interpolation formula we get

$$y_{30} = 17.1070 + (0.25) \times (-0.7638) + \frac{(0.25)(-0.750)}{2}(-0.0564) + \frac{(1.25)(0.125)(-0.75)}{6}(-0.0076) + \frac{(1.25)(0.125)(-0.175)(-1.75)}{24} \times (-0.0022)$$

$\Rightarrow 16.9216$

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TEACHING PLAN (SYNOPSIS)

Month : January Subject : Mathematics

TOPIC : stirlings Bessels difference formula Paper : V

Hours Required	12
Learning Objectives	
Previous Knowledge to be reminded	
Topic Synopsis	stirling's formula, Bessels difference formula

* Gauss Backward interpolation theorem.

* problems on Backward interpolation theorem

* State & prove stirling's Differences formula.

Stm:- If $y=f(x)$ is a function which takes the values $y_{-2}, y_{-1}, y_0, y_1, y_2$ corresponding to the values of $x = x_0-2h, x_0-h, x_0, x_0+h, x_0+2h$ then.

$$y_u = y_0 + u\left[\frac{\Delta y_0+\Delta y_{-1}}{2}\right] + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\left[\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right] + \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2}$$

* problems stirling's formula.

* Bessel's Difference formula & theorem

* problems on Bessel's formula.

* $$y_u = \frac{1}{2}[y_0+y_1] + \left[u-\frac{1}{2}\right]\Delta y_0 + \frac{u(u-1)}{2}\frac{\Delta^2 y_{-1}+\Delta^2 y_0}{2!} + \frac{(u-\frac{1}{2})u(u-1)}{3!}\Delta^3 y_{-1}$$

Thrust areas	
Skill to be learnt by Student	Gauss forward formula.
Examples/Illustrations	S. chand
Additional Inputs	Guass Backward formula

Teaching Models used	Mathematical models
Teaching Aids used	Black board and chalk.
References cited	Ref by V. Venkateswara Das, BVSS Sarma, by Krishna murthy, B. Anjaraya Sastry
Student Activity planned after the teaching	Assignment, seminar
Activity planned outside classes	
Any other	Group discussion

* Everett's Difference formula.

* theorem :- If $y=f(x)$ is a function which takes the values $y_{-2}\ y_{-1}\ y_0\ y_1\ y_2$ corresponding to the values of $x = x_0-2h,\ x_0-h,\ x_0,\ x_0+h,\ x_0+2h$, then

$$y_u = vy_0 + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \cdots$$

$$\frac{v(v^2-1)(v^2-4)\cdots(v^2-k^2)}{(2k+1)!}\Delta^{2k} y_{-k} + uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \cdots$$

$$\frac{u(u^2-1)(u^2-4)\cdots(u^2-k^2)}{(2k+1)!}\Delta^{2k} y_{1-k} \quad \text{where } u=\frac{x-x_0}{h} \ \& \ v=1-u.$$

* problems on Everett's formula & Given sums.

* Exercise 2.1 sums given.

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TEACHING PLAN (SYNOPSIS)

Month : January Subject : Mathematics

TOPIC : syllabus Paper : 6(A) Numerical method

Hours Required	12
Learning Objectives	mathematical Calculations
Previous Knowledge to be reminded	Guass forward.
Topic Synopsis	Newton, Backword.

Unit-2A

1. Interpolation with un Evenly spaced points,
2. Divided differences
3. properties of divided differences
4. Newton's divided difference formula.

Unit-3

1. Derivatives using Newton's forward differces formula, Newton's Backword differces formula.
2. Derivatives using Central differences formula. Stirling's interpolation formula.
3. Newton's divided differces formula, maximum and minimum values of a tabulated function.

Thrust areas	
Skill to be learnt by Student	Calculate the Guass forwards & Backword
Examples/Illustrations	S.chand
Additional Inputs	Guass forword & Backword formula's

Teaching Models used	mathematical models
Teaching Aids used	BlackBoard and chalk
References cited	Ref:- by V. Venkateswararao
Student Activity planned after the teaching	Assignment, Seminar.
Activity planned outside classes	Group discussion.
Any other	Quiz

Unit-4

Numerical integration

1. General quadrature formula.

Trapezoidal rule.

2. Simpson's 1/3 rule,

Simpson's 3/8 rule, & weddle's rules

3. Euler - Mclaurin formula for summation and quadrature,

The Euler transformation.

Unit-5

Numerical solution of ordinary Differential Equation

1. Introduction, solution by Taylor's series
2. picard's method of successive approximations.
3. Euler's method, Modified Euler's method, Runge-kutta method.

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TEACHING PLAN (SYNOPSIS)

Month : January

Subject : mathematics

TOPIC : mathematical spcical functions

Paper : Paper-II 7(A) mathematical special functions

Hours Required	12
Learning Objectives	mathematical Calculations
Previous Knowledge to be reminded	Beta function
Topic Synopsis	Beta & Gamma functions

unit-1

1. Euler's Integrals - Beta & Gamma functions

Beta & Gamma functions,

Beta function:-

If $m>0$, $n>0$ then Beta function $B(m,n)$ is defined as

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Note:-

* $B(m,n)$ is a symmetric function

i.e $B(m,n) = B(n,m)$

* $$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$= \int_0^1 (1-x)^{m-1}(1-(1-x))^{n-1}\,dx$$

$$= \int_0^1 x^{n-1}(1-x)^{m-1}\,dx$$

Thrust areas	—
Skill to be learnt by Student	—
Examples/Illustrations	S.chand
Additional Inputs	

Teaching Models used	mathematical models
Teaching Aids used	Blackboard, and chalk
References cited	Ref by V. venkateswar Das
Student Activity planned after the teaching	Assignment, seminar
Activity planned outside classes	Quiz
Any other	Group discussion

find $B(3,2)$ Here $m=3, n=2$

$$B(m,n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$$

$$B(3,2) = \frac{(3-1)!\,(2-1)!}{(3+2-1)!} = \frac{2!\,1!}{4!} = \frac{2\times1}{4\times3\times2} = \frac{1}{12}$$

* Transformation of Gamma functions Another from the Beta function.

* Realations between Beta and Gamma functation.

1) $\int_0^\infty e^{-2x} x^7 dx$

Sol: $\int_0^\infty e^{-kx} x^{n-1} dx = \frac{\Gamma(n)}{k^n}$

$$\int_0^\infty e^{-2x} x^7 dx = \int_0^\infty e^{-2x} x^{8-1} dx$$

$$= \frac{\Gamma(8)}{2^8} = \frac{(8-1)!}{2^8}$$

$$= \frac{7!}{2^8}$$

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TEACHING PLAN (SYNOPSIS)

Month : January

Subject : mathematics

TOPIC : Mathematical special functions

Paper : paper-II mathematical special functions

Hours Required	12
Learning Objectives	mathematical Calculations
Previous Knowledge to be reminded	power series
Topic Synopsis	

unit-2

Power series & power series solution of order Differential Equations

1. Introduction summary of useful results,

* Power series, radius of convergence

* theorems on power series

If the power series $\Sigma a_n x^n$ is such that $a_n \neq 0$ for all n

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{r}$ then $\Sigma a_n x^n$ is convergent for $|x| < r$ and divergent for $|x| > r$.

Proof: $u_n = a_n x^n$ so that $u_{n+1} = a_{n+1} x^{n+1}$

Then $\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left|\frac{a_{n+1} x}{a_n}\right| = |x| \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|}{r}$ — (1)

∴ By D'Alembert's ratio test $\Sigma a_n x^n$ converges absolutely if $\frac{|x|}{r} < 1$, i.e. $|x| < r$. Also $\Sigma a_n x^n$ diverge $|x| > r$.

Thrust areas	–
Skill to be learnt by Student	–
Examples/Illustrations	S chand
Additional Inputs	Power series

Teaching Models used	Mathematical models
Teaching Aids used	BlackBoard and chalk.
References cited	-
Student Activity planned after the teaching	Assignment, seminar.
Activity planned outside classes	Quiz
Any other	Group discussion

Determine the interval convergence of the power series $\sum \frac{(-1)^{n+1}}{n}(x-1)^n$

Let the given series be denoted by $\sum a_n (x-x_0)^n$

then we have $a_n = \frac{(-1)^{n+1}}{n}$ and $a_{n+1} \frac{(-1)^{n+2}}{n+1}$

$\therefore r =$ radius of convergence $= \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \left|-\frac{n+1}{n}\right| = 1$

Since the given power series is about the point $x = x_0 = 1$ the interval of convergence is

$x_0 - r < x < x_0 + r$, ie, $-1+1 < x < 1+1$, i.e, $0 < x < 2$

For $x = 2$, the given series reduces to the alternating series

$$\sum \frac{(-1)^{n-1}}{n} = \sum (-1)^{n-1} a_n \text{ say}).$$

$\rightarrow$ ST, $x = 0$ is an irregular singular point and $x = -1$ is a regular singular point of $x^2(x+1)^2 y'' + (x^2-1)y' + 2y = 0$

Sol: Dividing the equation $x^2(x+1)^2 y'' + (x^2-1)y' + 2y = 0$ by $x^2(x+1)^2$ we get

$$\frac{d^2y}{dx^2} + \frac{x-1}{x^2(x+1)}\frac{dy}{dx} + \frac{2}{x^2(x+1)^2} y = 0$$

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TEACHING PLAN (SYNOPSIS)

Month : February

Subject : Mathematics

TOPIC : power series

Paper : Paper-II mathematicalspecialtheory

Hours Required	12
Learning Objectives	power series
Previous Knowledge to be reminded	Mathematical Calculations
Topic Synopsis	power series solution about the ordinary

$(x+1)P(x) = \frac{x-1}{x^2}$ and $(x+1)^2 Q(x) = \frac{2}{x^2}$

$\Rightarrow (x+1)P(x)$ and $(x+1)^2 Q(x)$ are analytic at $x=-1$

Power series solution About the ordinary point $x=x_0$

Let the given Equation $y'' + P(x)y' + Q(x)y = 0$ — ①

If $x=x_0$ is an ordinary point of ① then ① has two non-trivial linearly independent power series solutions of the form $\sum_{n=0}^{\infty} a_n(x-x_0)^n$ — ②;

these power series converge in some interval

Thrust areas	–
Skill to be learnt by Student	
Examples/Illustrations	S.chand
Additional Inputs	Power series

Teaching Models used	mathematical models
Teaching Aids used	Black Board, chalk
References cited	–
Student Activity planned after the teaching	Seminar, Quiz
Activity planned outside classes	Quiz
Any other	Group discussion

③. $x^2y'' + xy' + (x^2 - n^2)y = 0$ at $x=0$ & $x = \infty$

sol: singularity at $x=0$, Given equation $x^2y''+xy'+(x^2-n^2)y=0$

$$\Rightarrow y'' + \left(\frac{1}{x}\right)y' + \left(\frac{x^2-n^2}{x^2}\right)y = 0 \quad \text{—①}$$

Comparing ① with $y'' + P(x)y' + Q(x)y = 0$,

we get $P(x) = \frac{1}{x}$ and $Q(x) = \frac{x^2-n^2}{x^2}$

Now $P(x)$ & $Q(x)$ are undefined at $x=0$ and so

they are not analytic at $x=0$

Hence $x=0$ is a singular point

Here $(x-0)P(x) = 1$ and $(x-0)^2 Q(x) = x^2 - n^2$

Now $(x-0)P(x)$ and $(x-0)^2 Q(x)$ are analytic at $x=0$

therefore $x=0$ is a regular singular point

singularity at $x=\infty$ Put $x = \frac{1}{t}$ then $t = \frac{1}{x}$ & $\frac{dt}{dx} = \frac{-1}{x^2}$ —②

Now $y' = \frac{dy}{dx} = \frac{dy}{dt} = \frac{dy}{dt}\left(\frac{-1}{x^2}\right) = -t^2\frac{dy}{dt}$ —③

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[-t^2\frac{dy}{dt}\right]\left[-\frac{1}{x^2}\right]$$

$$= \left[-t^2\frac{d^2y}{dt^2} - 2t\frac{dy}{dx}\right]\times(-t^2)$$

$$= t^4\frac{d^2y}{dt^2} + 2t^3\frac{dy}{dt} \quad \text{—④}$$

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TEACHING PLAN (SYNOPSIS)

Month : February | Subject : Mathematics

TOPIC : Hermite polynomials | Paper : Paper-II mathematical special functions

Hours Required	12
Learning Objectives	
Previous Knowledge to be reminded	power series
Topic Synopsis	power series solution

using ③ & ④ the Equation

$$\frac{1}{t^2}\left[t^4\frac{d^2y}{dt^2}+2t^3\frac{dy}{dt}\right]+\frac{1}{t}\left[-t^2\frac{dy}{dt}\right]+\left[\frac{1}{t^2}-n^2\right]y=0$$

$$\Rightarrow t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+\frac{1-n^2t^2}{t^2}y=0$$

$$\Rightarrow \frac{d^2y}{dt^2}+\frac{1}{t}\times\frac{dy}{dt}+\frac{1-n^2t^2}{t^4}=0 \quad ⑤$$

Comparing ⑤ with $\frac{d^2y}{dt^2}+p(t)\frac{dy}{dt}+Q(t)y=0$

we get $p(t)=\frac{1}{t}$ and $Q(t)=\frac{1-n^2t^2}{t^4}$

Then we have $(t-0)\,p(t)=1$ & $(t-0)^2Q(t)=\frac{1-n^2t^2}{t^2}$

Since $(t-0)^2Q(t)$ is not analytic at $t=0$, so $t=0$ is irregular singular point of ⑤

$\therefore x=\infty$ is an irregular singular point of the given Equation.

Thrust areas	–
Skill to be learnt by Student	–
Examples/Illustrations	S. chand.
Additional Inputs	legener's Equation

Teaching Models used	Mathematical, model
Teaching Aids used	Black Board and chalk
References cited	References Books
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside classes	Quiz
Any other	Group discussion

Unit - 3. HERMITE POLYNOMIALS

Def:- Hermite Differential Equation:-

The differential equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2\lambda y = 0$, where λ is a constant is called Hermite's differential equation.

* Solutions of Hermite's equation
* Hermite's polynomials
* Generating function for Hermite polynomials
* other forms for Hermite polynomials
* Rodgirques formula for Hermite polynomials

Theorem:- $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$

Proof:- $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$

$$\Rightarrow e^{x^2-(t-x)^2} = \frac{H_0(x)}{0!}t^0 + \frac{H_1(x)}{1!}t + \frac{H_2(x)}{2!}t^2 + \cdots + \frac{H_n(x)}{n!}t^n + \frac{H_{n+1}(x)}{(n+1)!}t^{n+1} + \cdots$$

Differentiating both sides partially w.r.t 't' 'n' times and then Putting $t=0$, we have

$$\left[\frac{\partial^n}{\partial t^n} e^{-(t-x)^2}\right]_{t=0} e^{x^2} = \frac{H_n(x)}{n!} n!$$

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TEACHING PLAN (SYNOPSIS)

Month : march

Subject : Mathematics

TOPIC : legender's Equations

Paper : paper-II

Hours Required	12
Learning Objectives	
Previous Knowledge to be reminded	Mathematical Calculations
Topic Synopsis	Hermite polynomials

Now $t-x=u$ i.e at $t=0$, $x=-u$

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial u} \text{ and } \left[\frac{\partial^n}{\partial t^n}e^{-(t-x)^2}\right]=\frac{\partial^n}{\partial u^n}(e^{-u^2})$$

$$=(-1)^n\frac{\partial^n}{\partial x^2}e^{-x^2}$$

$$=(-1)^n\frac{d^n}{dx^n}(e^{-x^2})$$

$$\therefore H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$$

* orthogonal properties of Hermite polynomials

* Recurrence formula for Hermite Polynomial

* Prove that $m<n$ then $\frac{d^m}{dx^m}\{H_n(x)\}=\frac{2^m n!}{(n-m)!}H_{n-m}(x)$

Sol: we have $\sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x)=e^{-t^2+2tx}$ —— ①

$$\therefore \sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{d^m}{dx^m}\{H_n(x)\}=\frac{d^m}{dx^m}e^{-t^2+2tx}=(2t)^m e^{-t^2+2tx}$$

$$=(2t)^m\sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x)$$

$$=2^m\sum_{n=0}^{\infty}\frac{1}{n!}t^{n+m}H_n(x)=2^m\sum_{r=m}^{\infty}\frac{1}{(r-m)!}t^r H_{r-m}(x)$$

Thrust areas	
Skill to be learnt by Student	Hermite polynomial
Examples/Illustrations	S.chand
Additional Inputs	Legender's Equation

Teaching Models used	Mathematical model
Teaching Aids used	Black Board
References cited	-
Student Activity planned after the teaching	Assignment Seminar
Activity planned outside classes	Quiz
Any other	Group discussion

Putting $n+m=r$, $n=r-m$ for $n=0, r=m$, for $n=\infty, r=\infty$

Equating the coefficient of t^n from the two sides we have

$$\frac{1}{n!}\frac{d^m}{dx^m}\{H_n(x)\} = 2^m \frac{1}{(n-m)!} H_{n-m}(x) \Rightarrow \frac{d^m}{dx^m}\{H_n(x)\}$$

$$= \frac{2^m n!}{(n-m)!} H_{n-m}(x).$$

Unit-4

Legendre's Equation

The differential Equation of the form

$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ is called

Legendre's differential Equation (or) Legendre's Equation)

where n is a constant.

Note:- Legendre's Equation can also be written as

$$\frac{d}{dx}\left\{(1-x^2)\frac{dy}{dx}\right\} + n(n+1)y = 0$$

* Solution of Legendre's Equation
* generating function of Legendre polynomials
* orthogonal properties of Legendre's polynomials

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TEACHING PLAN (SYNOPSIS)

Month : March Subject : Mathematics

TOPIC : legender's Equation Paper : mathematical special functions

Hours Required	12
Learning Objectives	-
Previous Knowledge to be reminded	Mathematical Calculation
Topic Synopsis	legender's Equation

* Recurrence formulae

theorem:- $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$

Proof:- we have $(1-2xh+h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n (P_n(x))$

Differentitating both sides w.r.t 'h' we have

$$-\frac{1}{2}(1-2xh+h^2)^{-3/2}(-2x+2h) = \sum_{n=0}^{\infty} nh^{n-1} P_n(x)$$

$$\Rightarrow (x-h)(1-2xh+h^2)^{-1/2} = (1-2xh+h^2)\sum_{n=0}^{\infty} nh^{n-1} P_n(x)$$

$$\Rightarrow (x-h)\sum_{n=0}^{\infty} h^n P_n(x) = (1-2xh+h^2)\sum_{n=0}^{\infty} nh^{n-1} P_n(x)$$

$$\Rightarrow (x-h)[P_0(x) + hP_1(x) + \cdots + h^{n-1}P_{n-1}(x) + h^n P_n(x) + \cdots]$$

$$= (1-2xh+h^2)[P_1(x) + 2hP_2(x) + \cdots + (n-1)h^{n-2}P_{n-1}(x) + nh^{n-1}P_n(x) + (n+1)h^n P_{n+1}(x) + \cdots] \quad \text{—(1)}$$

Equating the coefficient's of h^n from two sides, we have

$$xP_n(x) - P_{n-1}(x) = (n+1)P_{n+1}(x) - 2xnP_n(x) + (n-1)P_{n-1}(x)$$

$$\Rightarrow (2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

In $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$

Thrust areas	
Skill to be learnt by Student	-
Examples/Illustrations	S.chand
Additional Inputs	legender's Equation

Teaching Models used	Mathematical Model
Teaching Aids used	BlackBoard and chalk
References cited	–
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside classes	Quiz
Any other	Group discussion

* Christoffel's summation

* Rodrigue's formula

* Some $\frac{P_{n+1}-P_{n-1}}{2n+1} = \int P_n dx + c$

* $\int_{-1}^{+1} (x^2-1) P_{n+1} P_n' dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$

* (i) $\int_{-1}^{+1} P_n(x)\, dx = 0,\ n \neq 0$ and (ii) $\int_{-1}^{+1} P_0(x)\, dx = 2.$

Sol: from Rodrigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$

$\therefore \int_{-1}^{+1} P_n(x) = \frac{1}{2^n n!} \int_{-1}^{+1} \frac{d^n}{dx^n} (x^2-1)^n dx = \frac{1}{2^n n!} \left\{ \frac{d^{n-1}}{dx^{n-1}} (x^2-1)^n \right\}_{-1}^{+1}$ —(1)

Now $\frac{d^{n-1}}{dx^{n-1}} (x^2-1)^n = \frac{d^{n-1}}{dx^{n-1}} (x+1)^n (x-1)^n$

$= (x+1)^n \frac{d^{n-1}}{dx^{n-1}} (x-1)^n + (n-1) n (x+1)^{n-1} \frac{d^{n-2}}{dx^{n-2}} (x-1)^n + \cdots + (x-1)^n \frac{d^{n-1}}{dx^{n-1}} (x+1)^n$

$= (x+1)^n \frac{n!}{1!} (x-1) + n(n-1)(x+1)^{n-1} \frac{n!}{2!} (x-1)^2 + \cdots + (x-1)^n n! (x+1)$

$= 0$ when $x = -1$ or 1, since each term $(x-1)$ & $(x+1)$

$\therefore \int_{-1}^{+1} P_n(x) dx = 0$

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TEACHING PLAN (SYNOPSIS)

Month : March Subject : Mathematics

TOPIC : Bessel's Equations Paper : paper-II mathematical special functions

Hours Required	12
Learning Objectives	—
Previous Knowledge to be reminded	
Topic Synopsis	Bessel's Equations

Unit-5

BESSEL'S EQUATIONS

* Def :- the differentional equation of the form.

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left[1 - \frac{n^2}{x^2}\right]y = 0$$ is called Bessel's differential Equation (or) Bessel's Equation.

* solutions of Bessel's Differential Equation.

* If $a_0 = \frac{1}{2^n \Gamma(n+1)}$ then the solution of the Bessel's Equation

$$y = a_0 x^n \left[1 + (-1)\frac{x^2}{2^2 1! (n+1)} + (-1)^2 \frac{x^4}{2^4 2! (n+1)(n+2)} + \cdots\right]$$

Bessel's function.

* General solution of Bessel's Equation.

* Recurrence formula for $J_n(x)$

$$x J_n'(x) = n J_n(x) - x J_{n+1}(x)$$

Thrust areas	—
Skill to be learnt by Student	—
Examples/Illustrations	S.chand
Additional Inputs	Bessel's Equation.

Teaching Models used	Mathematical model
Teaching Aids used	Black Board
References cited	–
Student Activity planned after the teaching	Assignment
Activity planned outside classes	Quiz
Any other	Group discussion.

Theorem * $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

Proof:- Recurrence formula I & II

$$xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x) \quad \& \quad xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$$

adding we have $2xJ_n'(x) = x[J_{n-1}(x) - J_{n+1}(x)]$

Hence $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \frac{1}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

$$2J_n'(x) = \sum_{r=0}^{\infty} (-1)^r \frac{2}{r!\,\Gamma(n+r+1)} (n+2r)\frac{1}{2}\left(\frac{x}{2}\right)^{n+2r-1}$$

$$= \sum_{r=0}^{\infty} (-1)^r \frac{n+r+r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1}$$

$$= \sum_{r=0}^{\infty} (-1)^r \frac{n+r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1} + \sum_{r=0}^{\infty} (-1)^r \frac{r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1}$$

$$= \sum_{r=0}^{\infty} (-1)^r \frac{1}{r!\,\Gamma(n+r)} \left(\frac{x}{2}\right)^{n+2r-1} - \sum_{r=0}^{\infty} (-1)^{r-1} \frac{1}{(r-1)!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1}$$

$$= \sum_{r=0}^{\infty} (-1)^r \frac{1}{r!\,\Gamma(n-1+r+1)} \left(\frac{x}{2}\right)^{n-1+2r} - \sum_{s=0}^{\infty} (-1)^s \frac{1}{s!\,\Gamma(1+n+s+1)} \left(\frac{x}{2}\right)^{n+1+2s}$$

$$= J_{n-1}(x) - J_{n+1}(x)$$

Hence $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

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TEACHING PLAN (SYNOPSIS)

Month : March   Subject : mathematics

TOPIC : Bessels Equations   Paper : mathematical special function

Hours Required	12
Learning Objectives	—
Previous Knowledge to be reminded	
Topic Synopsis	Bessel's Equations

* $\frac{d}{dx}[x^{-n} J_n(x)] = x^{-n} J_{n+1}(x)$

Proof :- $\frac{d}{dx}[x^{-n} J_n(x)] = -n x^{-n-1} J_n(x) + x^{-n} J_n'(x) = x^{-n-1}[-nJ_n(x) + xJ_n'(x)]$

$= x^{-n-1}[-nJ_n(x) + \{nJ_n(x) - xJ_{n+1}(x)\}]$

$= x^{-n-1}[-xJ_{n+1}(x)]$

$= -x^{-n} J_{n+1}(x)$

Hence $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

* $\frac{d}{dx}[x^{n} J_n(x)] = x^{n} J_{n-1}(x)$

* $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)} = \sum_{-\infty}^{\infty} z^n J_n(x)$

* $\frac{d}{dx}[J_n^2 + J_{n+1}^2] = 2\left[\frac{n}{x} J_n^2 - \frac{n+1}{x} J_{n+1}^2\right]$

* $\frac{d}{dx}\left[\frac{J_{-n}}{J_n}\right] = \frac{-2\sin n\pi}{\pi x J_n^2}$

* $x^n J_n(x)$ is a solution of $x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0$

Thrust areas	
Skill to be learnt by Student	
Examples/Illustrations	S. chand
Additional Inputs	

Teaching Models used	Mathematical model
Teaching Aids used	Black Board
References cited	-
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside classes	Quiz
Any other	Group discussion

* show that $y = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\phi)\,d\phi$

Sol, Let $y = J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\phi)\,d\phi$ — ①

$$\frac{dy}{dx} = \frac{-1}{\pi}\int_0^{\pi} \sin(x\sin\phi)\sin\phi\,d\phi \quad — ②$$

$$\frac{d^2y}{dx^2} = \frac{-1}{\pi}\int_0^{\pi} \cos(x\sin\phi)\sin^2\phi\,d\phi \quad — ③$$

Evaluating the R.H.S of eqn ② by the method of integrating by parts we have

$$\frac{dy}{dx} = \frac{-1}{\pi}\left[-\sin(x\sin\phi)\cos\phi\Big]_0^{\pi} + \int_0^{\pi} \cos(x\sin\phi)\,x\cos^2\phi\,d\phi\right]$$

$$= \frac{-x}{\pi}\int_0^{\pi} \cos(x\sin\phi)\cos^2\phi\,d\phi = \frac{-x}{\pi}\int_0^{\pi} \cos(x\sin\phi).(1-\sin^2\phi)\,d\phi$$

$$= \frac{-x}{\pi}\int \cos(x\sin\phi)\,d\phi + \frac{x}{\pi}\int_0^{\pi} \cos(x\sin\phi)\sin^2\phi_0$$

$$= -xy - x\frac{d^2y}{dx^2} \text{ from ① \& ③}$$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + y = 0$$ which is Bessels Equation for $n=0$

Hence $y = \frac{1}{x}\int_0^{\pi} \cos(x\sin\phi)\,d\phi$ satisfies the

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TEACHING PLAN (SYNOPSIS)

Month : March

Subject : Mathematics

TOPIC : Group theory

Paper : I (ABSTRACT ALGEBRA)

Hours Required	12
Learning Objectives	
Previous Knowledge to be reminded	
Topic Synopsis	Group theory

Unit-I

GROUPS

Def:- Let A be a non empty set Any mapping from $A \times A$ into 'A' is called binary operation (or) binary composition on 'A'.

binary operation :- let '$*$' be an binary operation on an nonempty set A and $B \subseteq A$. The B is said to be closed with respect to the binary operation $*$ if $x*y \in B \; \forall \; x, y \in B$

Types of Binary operation

A binary operation $*$ on a non empty set A is said to be associative if $(x*y)*z = x*(y*z) \; \forall \; x, y, z \in A$

A binary operation $*$ on non empty set A is said to be commutative if $x*y = y*x \; \forall \; x, y \in A$

which of the following binary operation on N, binary operation are associative & commutative.

① $a*b = a+b$

② $a*b = a-b$

Thrust areas	
Skill to be learnt by Student	
Examples/Illustrations	
Additional Inputs	

Teaching Models used	Mathematical model
Teaching Aids used	Black Board
References cited	-
Student Activity planned after the teaching	Assignment, Seminar
Activity planned outside classes	Quiz
Any other	Group discussion

* finite Groups and infinite groups

Theorem: A finite semigroup 'G' Satisfying Cancellation laws in a group

Proof: Since 'G' is a finite, let $G = \{a_1, a_2 ---- a_n\}$

let $a \in G$. write $aG = \{aa_1, aa_2 ---- aa_n\}$

let $x \in aG$

$x \in aG \Rightarrow x = aa_i$, where $a_i \in G$

$a \in G, a_i \in G \Rightarrow aa_i \in G \Rightarrow x \in G \quad \because aG \subseteq G$

if $aa_i = aa_j$ then by left Cancellations law $a_i = a_j$.

Thus $a_i \neq a_j \Rightarrow aa_i \neq aa_j$

$\therefore aG$ Contain 'n' different Elements of G and $aG \subseteq G$

Sin aG, G Contains the Same number of Elements, $aG = G$

Thus $aG = G \; \forall a \in G$

Illy $Ga = G \; \forall a \in G$

Let $a, b \in G$ Now $a \in G \Rightarrow Ga = G$ and $aG = G$

$b \in G \Rightarrow b \in Ga$, $b \in aG \Rightarrow b = ya$, $b = ax$ where $y \in G, x \in G$

$\therefore$ The Equations $ax = b$, $ya = b$ have Solutions in G

from theorem 1.2.7, 'G' is a group.

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